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Table 1. Related Work Packages status

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1 Introduction

At high temperature, refractory bricks and mortars have a non-linear behaviour (Task 3.2). To obtain a material that has a behaviour equivalent to that of the masonry, non-linear homogenisation methods are used. They will be adapted to the case of refractory masonries and applied to the materials used in large industrial structures. This report will review these different aspects.

2 Numerical simulation of masonry

As masonry structures are composed by blocks and joints, they feature a great variety in combinations of different component materials, geometry and textures, implying a great number of descriptive parameters, thus, presenting itself as a complex research field [1]. Hence, the importance of sophisticated numerical tools, capable of predicting the behavior of the structure from the linear stage, through cracking and degradation until complete loss of strength [2]. However, only in the 1990's did the masonry research community begin to show interest in sophisticated numerical tools to study masonry as an opposition to the prevailing tradition of rules-of-thumb and empirical formulae [2].
As a composite material, masonry has certain particularities that difficult the adoption of existing numerical tools from more advanced research fields, such as mechanics of concrete, rock and composite materials. Hence the necessity for different approaches. Due to the heterogeneous states of stress and strain on a masonry structure, it would be useful to assess the local behaviour of the masonry with mortar through a micro-model. The mechanical properties of both the unit and mortar and the interface between them could then be considered (Figure 1(a)). This approach, however, is only adequate for small structural elements. In case of large analyses, the knowledge of the interaction between units and mortar is, generally, negligible for the global structural behaviour. Therefore, a different approach, denoted macro-modelling, can be used. In this approach, the material is regarded as an anisotropic composite and a relation is established between average masonry strains and average masonry stresses (Figure 1(b)). This is a phenomenological approach, meaning that the material parameters must be performed in masonry tests of sufficiently large size under homogeneous states of stress. A complete macro-model must reproduce an orthotropic material with different tensile and compressive strengths along the material axes, as well as different inelastic behaviour for each material axis [3].

(a) (b)

Figure 1 - Modelling strategies for masonry structures [3]: (a) micro-modelling; (b) macro-modelling

Lourenço et al. [4] stated that an accurate micro-model must include all recognised failure modes that characterise masonry (Figure 2), such as;

- cracking along the joints,
- sliding along the joints,

- cracking of units in direct tension,
- development of friction at the interface with diagonal tension cracking of the units and
- crushing of masonry

According to the described phenomena, the first two modes are considered to be joint phenomena, the third one is a unit mode while the last two usually involve both the unit and joints. The approached followed by Lourenço et al. [4] is to concentrate all the damage in the relatively weak joints and in potential pure tension cracks in the units placed vertically in the middle of each unit.

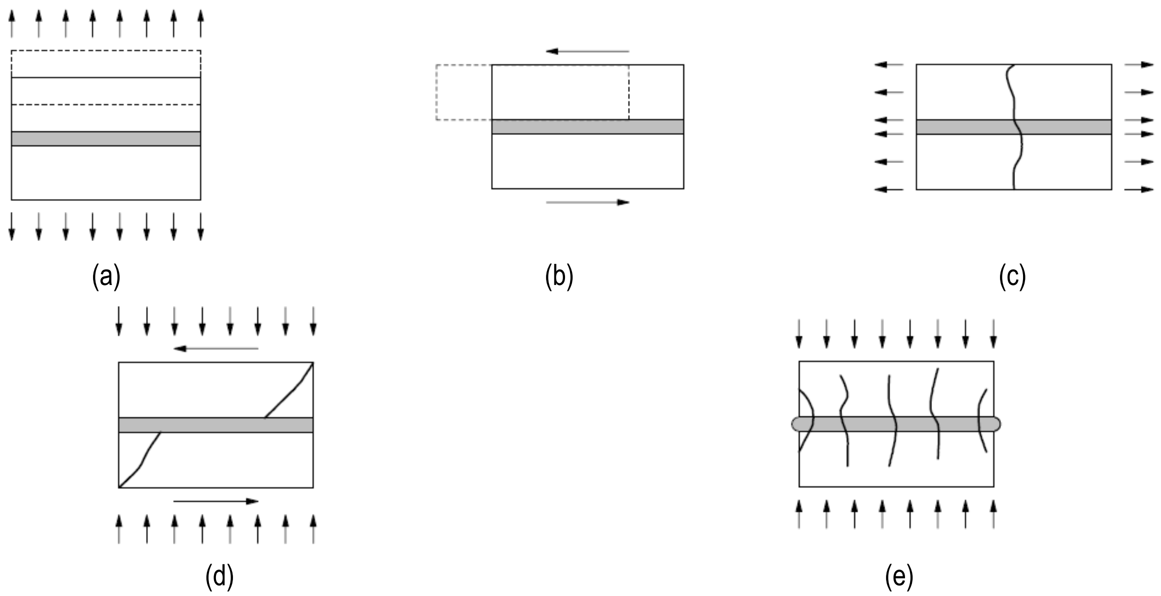

Figure 2 - Masonry failure modes [3]: (a) joint tension cracking; (b) joint slipping; (c) unit direct tension cracking; (d) unit diagonal tension cracking; (e) masonry crushing.

However, the use of micro-models is limited by the high computational cost and the requirement for accurate experimental data. The alternative is to use macro-models, in which the composite behaviour is described in terms of macro or average stress and strains. To implement this approach, there exist two possibilities. In one of them, it is necessary to gather, collate and interpret extensive experimental data and, ultimately, manipulate it in the form of master curves in terms of non-dimensionable variables. The results, however, are limited to the conditions under which the data are obtained, which might make necessary a different set of costly experimental programs, in case of new materials and/or different loading conditions. The other option is to seek the more fundamental approach of homogenisation techniques. This approach aims to describe the behaviour of the composite through the geometry and the behaviour of the representative volume element (or basic cell, see Figure 3), and thus grants a predictive capability [3].

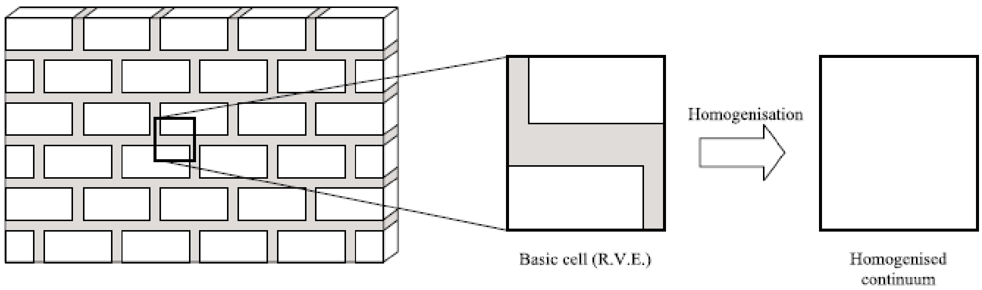

Figure 3 - Basic cell for masonry and objective of homogenisation [5].

According to Lourenço [5], as a closed-form solution of a homogenisation problem is hard to be obtained, three lines of action might be defined. The first is a very powerful mathematical approach that handles the brickwork structure of masonry by considering the salient features of the discontinuum within the framework of a generalised/Cosserat continuum theory [6], [7]. However, even though this technique demonstrates good responses, its inherent mathematical complexity means it has not been adopted by many researchers. The second approach [8]–[10], refers to the application of a single step homogenization method, which considers adequate boundary conditions and the exact geometry. Due to the complexity of the masonry basic cell, a numerical solution of the problem is necessary, which is obtained using the finite element method. Anthoine [8], [9] and Urbanski

[10] applied this theory to determine macro-parameters of masonry and to avoid carrying out analysis at the structural level. If the rigorous application of the homogenisation theory for the non-linear behaviour of the complex masonry basic cell is enforced, it implies solving the problem for all possible macroscopic loading histories, since the superposition principle does not apply anymore. Thus, the complete determination of the homogenised constitutive law would require an infinite number of computations. The third approach might be defined as an engineering approach, since the objective is to replace the complex geometry of the basic cell by a simplified geometry so that a closed-form solution of the homogenisation problem is possible. Considering this objective, Pande et al. [11], Maier et al. [12] and Pietruszczak and Niu [13] introduced homogenization techniques in an approximate manner. A two-step homogenization method has been commonly performed, with head (or vertical) and bed (or horizontal) joints being introduced successively. Hence, the masonry can be assumed to be a layered material, which simplifies the problem significantly.

2.1 DEVELOPMENT OF THE HOMOGENIZATION APPROACH

Through the application of macro-models and homogenization techniques, it is possible to ascertain the behaviour of masonry structures more broadly. Considering the engineering approach, where the complex basic cell geometry is replaced by a simplified one, is the most straightforward method, this section is dedicated to the development and evolution of these type of models.

In Lourenço [3], a macro-model based on an anisotropic plasticity model was presented, introducing further development to the two-step homogenisation procedure in the form of a new matrix formulation. This allowed a clearer implementation of linear elastic homogenisation algorithms and, enabled the implementation of a relatively simple extension to non-linear behaviour. However, the regular offset of vertical mortar joints belonging to two consecutive layered unit courses is not explicitly accounted for using the two separate homogenisation steps. Also, the result depends on the order in which the two steps are carried out.

Lourenço and Rots [14] evaluated the performance of an interface elastoplastic constitutive model for the analysis of unreinforced masonry structures. A rational unit-joint model able to describe cracking, slip, and crushing of the material was applied. The material behaviour considered softening plasticity for tension, shear and compression, with a consistent treatment of the intersections defined by these modes. The numerical implementation was based on local and global Newton-Raphson methods, implicit integration of the rate equations and consistent tangent stiffness matrices. The parameters required to define the model were derived from experiments in units, joints, and small masonry samples. Finally, the model was applied to analyse masonry shear-walls. It was concluded that the model was able to predict the experimental collapse load and behaviour accurately. However, it was shown that if there are large differences of stiffness (>10) between unit and mortar, large errors can occur in the standard two-step homogenisation technique.

To address this problem Lourenço and Zucchini [5] presented a micro-mechanical homogenisation model for masonry, including additional deformation modes of the basic cell in relation to the two-step homogenization model. The representative volume element for this one-step homogenization model is illustrated in Figure 4. The main problems in the two-step homogenization process are the errors caused by the difference in stiffness between the mortar and the bricks, which are heightened in case of a non-linear analysis. Thus, with the one-step homogenization model proposed by the authors, they were able to demonstrate that the accuracy of the model assessed for an increasing ratio between the stiffness of the two components was better than for the two-step one. Moreover, it was shown that the anisotropic failure surface obtained from the proposed micromechanical model, assuming elastic–brittle behaviour of unit and mortar, seemed able to, qualitatively, reproduce the experimental results available for the composite behaviour of masonry. The quantitative assessment of the model was not addressed due to the reduced experimental data available.

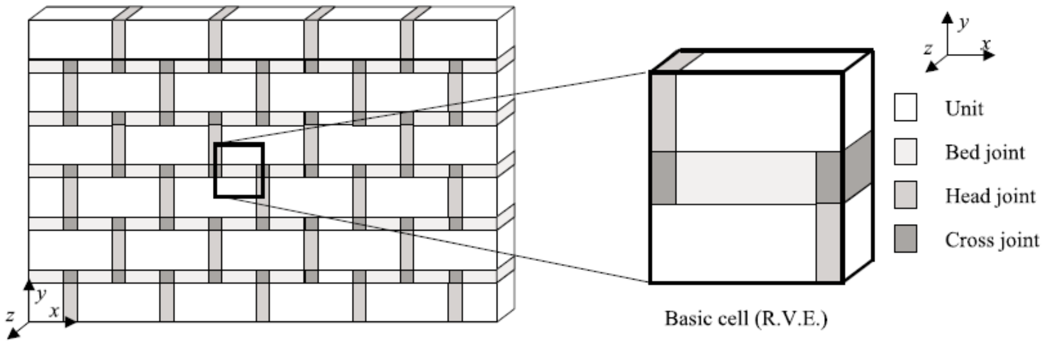

Figure 4 - Definition of masonry axes and masonry components considered in the analysis: unit, head joint, bed joint and cross [5].

2.2 MASONRY IN COMPRESSION

As masonry has a low tensile strength, cracks are responsible for the majority of its non-linear behaviour. Masonry features orthotropic behaviour with material axes normal and parallel to the bed joints, which causes the response to be straightforward for tension normal to the bed joints and rather complex for tension parallel to the bed joints. Hence, Zucchini and Lourenço [15], addressed the formulation and implementation of a coupling between a micro-mechanical homogenization model and an isotropic damage model for masonry components. An improved additional deformation mode is considered regarding a homogenisation technique developed in Lourenço and Zucchini [5]. The simulation has been accomplished by coupling the elastic micro-mechanical model with a scalar damage model for joints and units and employing an iterative solution procedure to calculate the damage coefficients. Finally, the model was validated by comparing numerical results available in the literature, using interface modelling masonry in compression

Unreinforced masonries are characterised by poor mechanical performance in tension, due to a very low tensile strength. As such, masonry structures have been employed only in compression, unless reinforced or prestressed masonry is used [16]. Therefore, the compressive strength of masonry in the direction normal to the bed joints has been traditionally regarded as the sole relevant structural material property, at least until numerical methods for masonry structures were introduced [16]. With the study of stacked masonry under compression, simple compression failure theories were developed, that later implied that the difference in elastic properties of the unit and mortar is a precursor of failure. Also, as uniaxial compression of masonry was proved to lead to a state of triaxial compression in the mortar and of compression/biaxial tension in the unit, the application of sophisticated non-linear analyses of masonry under compression became necessary to reproduce the experimental response of the masonry [16]. Hence, in this section, some works that address masonry in compression are presented.

Zucchini and Lourenço [17], have proposed a homogenisation technique to simulate the behaviour of masonry in compression. Starting from a simple linear optimisation of the stress field, the problem was derived on the elementary cell in other to find the homogenised failure surface of masonry. From there, four other models with more accurate optimisation were presented. The model was validated through comparisons with experimental data and kinematic approaches, showing itself able to reproduce homogenised failure surfaces of masonry varying both the geometrical characteristics of the cell and the mechanical properties of the components. In the accompanying paper [18] the homogenised failure surfaces obtained in Zucchini and Lourenço [17] were implemented in a finite element limit analysis code and relevant structural examples were treated both with a lower and an upper bound approach.

Zucchini and Lourenço [16] contributed to the understanding of masonry under compression, using a non-linear homogenisation tool that includes tensile damage and compressive plasticity. An iterative procedure was adopted to solve the simplified homogenised model, which considered a few ingenious micro-deformation mechanisms, and was able to accurately reproduce complex simulations carried out with non-linear continuum finite element analysis, at a marginal cost of CPU time and with no convergence difficulties. Furthermore, a comparison between the homogenised approach and experimental results of masonry under compression was made and indicated that an estimation of the compressive strength of masonry better than the one provided by the codes was possible, using the mechanical and geometrical properties of the masonry components. The model presented was based on a homogenization approach previously developed by the authors [5], [15], which was extended for the first time to the case of masonry under compression.

In Bertolesi et al [19], two simple homogenization models suitable for the non-linear analysis of masonry walls in-plane loaded were presented. A rectangular running bond elementary cell was discretized using twenty-four constant stress three-noded plane-stress triangular elements and linear two-noded interfaces (Figure 5). Since non-linearity is concentrated on mortar reduced to the interface, it exhibits a holonomic behaviour with softening. Additionally, the authors showed how the mechanical problem in the unit cell might be characterized using a few displacement/stress variables and how homogenized stress-strain behaviour can be evaluated through a small-scale system of non-linear equations. This means that, at a structural level, it is not necessary to solve a homogenization problem at each load step in each Gauss point and a direct implementation into commercial software as an external user-supplied subroutine is straightforward. Non-linear structural analyses were conducted on a variety of different problems, for which experimental and numerical data are available in the literature, in order to show that accurate results can be obtained with a limited computational effort.

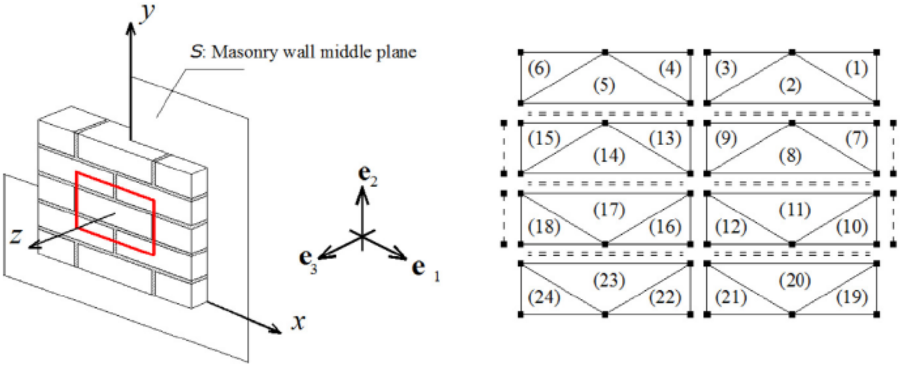

Figure 5 - Masonry pattern analysed and the subdivision of the RVE into 24 CST triangular elements and 1/4 of the RVE into 6 elements [19].

2.3 DRY JOINTS / MORTARLESS JOINTS

Research on structural masonry has been mostly devoted to the characterization of mortared-joint masonry, however, as the wear lining of the steel ladle is built with mortarless masonry, it is important to understand the complex behaviour of dry joints masonries.

As the behaviour of dry joints under cyclic loading is a key aspect for the study of seismic actions, Ramos and Lourenco [20] investigated the characterization of Coulomb failure criterion and the load-displacement behaviour of dry masonry joints under cyclic loading. This study included aspects such-as surface roughness, dilatancy and inelastic behaviour. In order to obtain the data necessary to characterize this behaviour, the authors used displacement controlled test set-up using masonry couplets. In addition to providing a basis for understanding the behaviour of masonry joints in tension, the experiments also contributed to the definition and parameterization of advanced non-linear numeric models. The experiments indicated that the coulomb friction law was adequately represented the behaviour of dry masonry joints under moderate stress levels. However, it is important to note that significant differences can be found between the initial and the final friction angle, after a given number of cycles. Concerning the numerical simulation of the hysteretic behaviour of dry masonry joints, it was shown that the model should include a hardening branch in the loading stage and that the shear deformation is fully plastic.

In Lourenço et al. [21], experimental results concerning the behaviour of dry joint masonry were presented. The most relevant points were related to the strength response of stone dry joint masonry walls subjected to in-plane combined compressive and shear loading. Remarkably, the walls exhibit a significant increase of stiffness with the amount of vertical compression provided to them. Additionally, the multi-surface interface numerical model proposed by Lourenço and Rots [14], stemming from plasticity and based on a micro-modelling strategy, was presented and discussed concerning its capacity to simulate the obtained experimental results. The model was calibrated with data collected from complementary tests carried out on specimens and prisms made of the same type of stone. The authors concluded that the numerical modelling enabled a detailed simulation of the response of the walls throughout the load process leading to failure and found that the prediction of the collapse loads was in accordance with the experiments. Finally, the application of a simplified method of analysis based on a continuum of diagonal struts was also addressed. It was concluded that despite its simplicity, the model was able to predict the ultimate loads while allowing a certain understanding of the ultimate conditions of the walls.

Senthivel et al [22] presented a numerical analysis to evaluate the seismic performance of dry stack masonry shear walls found in ancient masonry structures. A numerical simulation based on experimental test data was carried out to model the monotonic and reversed cyclic load-displacement hysteresis curves of dry-stack mortarless sawn stone masonry using a multi-surface interface model where stone units and joints were assumed elastic and inelastic, respectively. The stones were modelled using an eight node continuum plane stress elements with Gauss integration and the joints were modelled using a six node and zero thickness line interface elements with Lobatto integration proposed by Lourenço and Rots [14]. Elastic and inelastic parameters, as well as strength have been calculated based on the experimental test data.

2.4 NUMERICAL RESEARCH ON REFRACTORY MASONRY AT ELEVATED TEMPERATURE

Up to this point, none of the studies presented were directly related to refractory masonry. Hence, in this section, to get an insight about the common practices, regarding this application, some numerical research specific for refractories masonry is presented.

Gasser et al. [23], for example, proposed a solution for the homogenisation of masonry by studying the joint effects on the refractory lining. The thermomechanical loads were simulated on an elementary cell using a model developed at the scale of bricks and joints, where joints were represented as contacts (Figure 6). In order to obtain the parameters for the homogenised materials, the material behaviour was considered to be orthotopic, elastic and non-linear depending on joint opening and closure. Two simplifications were also assumed, the first one regarding the possible sliding between the bricks and in the second, the progressive joint closure was not considered. Furthermore, the parameters were determined by an inverse identification process and the developed model was validated by a thermomechanical test on an experimental structure containing flat masonry.

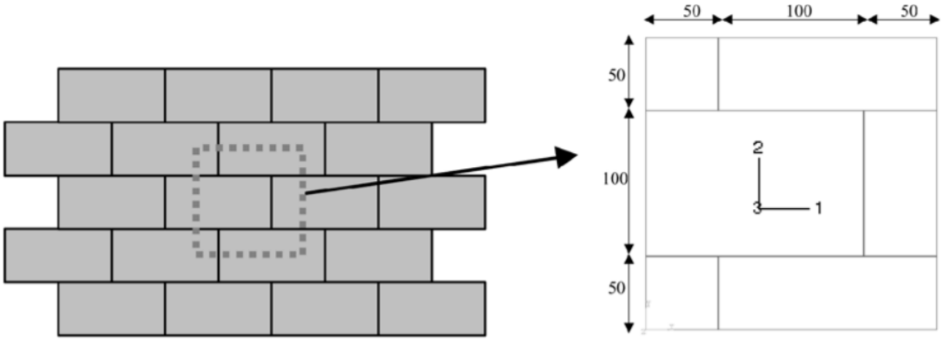

Figure 6 - Representative element cell [23].

Nguyen, T. M. H. et al.[24], extended the work done by Gasser, A. et al. [25] by conducting a study with a goal to propose an equivalent material that has the same behaviour as masonry. The proposed equivalent material has four different behaviours depending on the local joint state, each of them determined by homogenisation techniques (Figure 7). The transition criterion between the different joint states is based on the local unilateral contact conditions rewritten in terms of macroscopic strain and identified accordingly by numerical simulations. The developed model was validated with a biaxial compression test on a masonry unit, and it was found to be in good agreement with the experimental results. It was also observed that the non-linear behaviour of the masonry was mainly due to the existence of different joint states in the whole lining rather than to the non-linear crushing of contact surfaces of bricks.

Blond, E. *et al.* [26], further extended the work performed by Gasser, A. *et al.*[25] applying the periodic homogenization techniques previously developed to obtain the equivalent material properties. A simple hypothesis on the constituent material, considering elastic mortar and brick and discrete damage for mortar, with joint open or closed, was used. The model demonstrated good capabilities to capture the behaviour of masonry in two different applications.

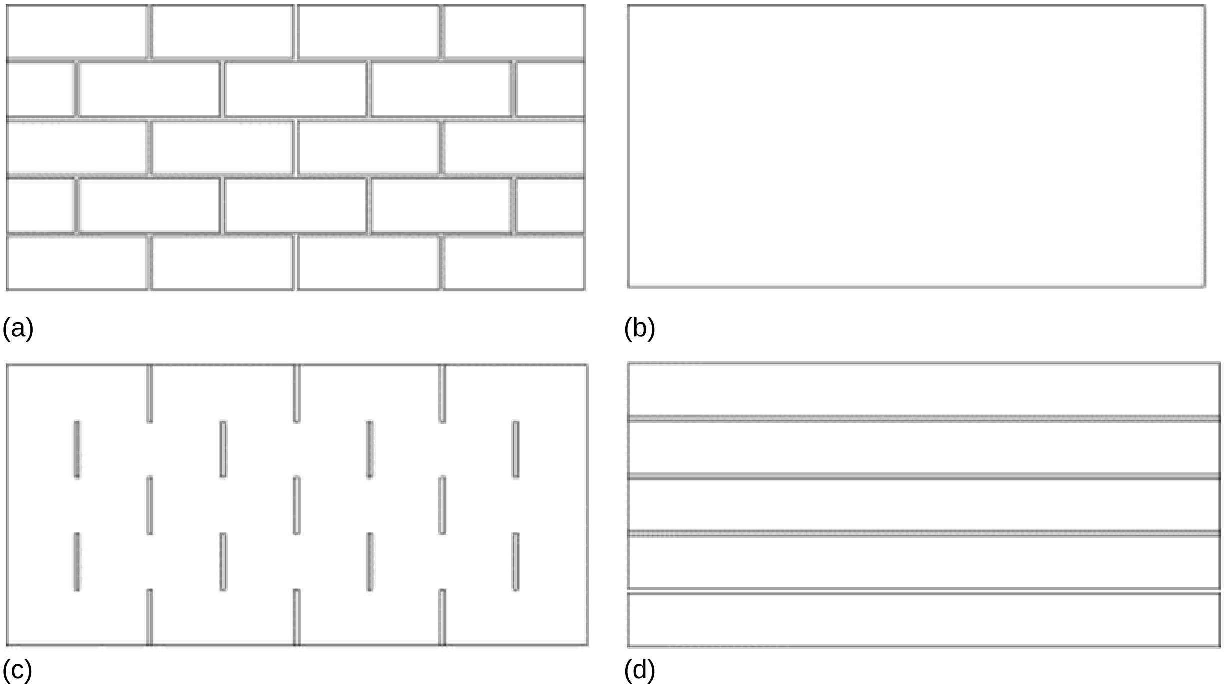

Figure 7 - Joint states [24]: a) joints open in both direction; b) joints closed in both direction; c) only head joints open; d) only bed joints open.

Rekik, A., Nguyen, T. T. N. and Gasser, A. [27], presented a multi-level modelling of viscoelastic microcracked masonry. In the finite element model, the mortar was assumed to be viscoelastic and microcracked while the bricks were assumed to be elastic and undamaged. The microcracks' distribution was assumed to be isotropic. The behaviour of the viscoelastic microcracked masonry was provided by two steps. The first one accounted for the effect of microcracks on the macroscopic deformation of the mortar and determines a linear relation between apparent macroscopic stress and strain. The second one was based on the coupling between asymptotic analysis and homogenisation theory applied for a periodic masonry. The proposed models provided analytical solutions for the effective behaviour of such structures, and therefore, allowed the prediction of mostly stressed and deformed areas in microcracked masonry structures.

Rekik, A. and Gasser, A. [28], used finite element models to predict the effective tangent properties of microcracked viscoelastic masonry. The authors identified the short and long terms of an approximate analytical creep curve for the mortar. And later, provided orthotropic overall properties of masonry by homogenisation techniques using finite element models. Additionally, an alternative model to an incremental homogenisation of masonry whit a high computational resource demand was presented. The case of a compressed wall in the long term was presented. Finally, the results provided by the finite element models might be considered as reference solutions enabling a rigorous assessment of analytical model proposed by other researchers.

2.5 THE MICRO-MACRO HOMOGENIZATION TECHNIQUE

Using a finite element based homogenization method, a micro-macro homogenization technique of refractory masonry with mortar joints has been developed at the University of Orléans by Landreau et al. [29] and Brulin *et al.* [30]. The technique allows the replacement of the bricks and mortar by an equivalent homogenous material and the decrease in structure stiffness due to damage of brick/mortar interface. For this purpose, four possible periodic joint patterns have been defined based on the state of bed and head mortar joints (damaged or undamaged) to be accounted for. Each joint pattern represents a different periodic masonry structure with different equivalent elastic behaviour. The four joint patterns are as follows (see Figure 8):

- Pattern 1: all bed and head mortar joints are closed (i.e. undamaged).
- Pattern 2: bed joints are open (damaged) and head joints are closed (undamaged).
- Pattern 3: bed joints are closed (undamaged) and head joints are open (damaged).
- Pattern 4: all mortar joints are open (damaged).

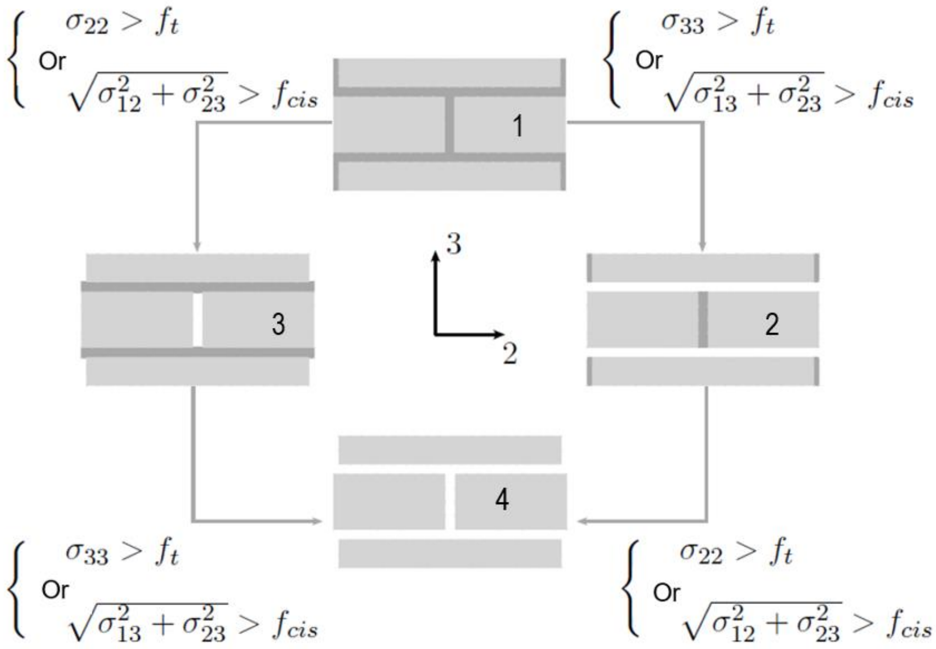

Figure 8 - Possible joint patterns of refractory masonry structure with mortar joints and joints opening criteria [30].

At high temperature, the constitutive material of the bricks was assumed to undergo small deformations and to exhibit an isotropic linear elasticity. Also, the mortar was assumed to obey isotropic linear elasticity and a yield surface in tension and in shear. The equivalent elastic properties of each joint pattern have been determined using finite element-based homogenization approach using a Representative Volume Element (RVE) subjected to homogeneous strain boundary conditions. Several finite element simulations of uniaxial, biaxial and shear loading have been performed. From the simulated combination of uniaxial, biaxial and shear tests, the strain energy has been computed. The knowledge of the applied macroscopic strain and the computed energy allows the calculation of the macroscopic stress and effective mechanical properties as described by Nguyen, T. M. H. *et al.* [31]. During operation, refractory masonry structure with mortar joints is subjected to mechanical or thermal loading and unloading. As a result, bed and/ or head joints may open. Therefore, the structure may change from one pattern to another leading to a change in the homogenized elastic behaviour. This change has been considered by using a suitable joint opening and pattern transition criterion written in terms of macroscopic stresses, ultimate tensile strength of brick/mortar interface, cohesion and friction angle as described in Figure 8. The developed model demonstrated good capabilities to capture the behaviour of masonry wall subjected to shear loading. In addition, it has been used to simulate and predict the homogeneous thermomechanical response of large scale industrial applications such as coke oven heating walls [29].

3 Numerical modelling of masonry structures with dry joints

Refractory masonry structures with dry joints are widely used in the steel industry for the linings of many high-temperature industrial applications including the steel ladle. Refractory masonry bricks with length (l_b), high (h_b), and width (w_b) are periodically arranged in running bond texture. Joints with thickness (g << l_b, h_b, w_b) separate the blocks from each other (see Figure 9). These joints are present due to surface roughness of the blocks, surface shape defaults and brick dimension variations. Two categories of joints are defined based on their orientation: bed joints with thickness g_{bed} (in the horizontal direction) and head joints with thickness g_{head} (in the vertical direction). Under thermal or mechanical loading/unloading, these joints can close and reopen leading to a change in the overall thermomechanical response of the refractory masonry structure. The influence of joint closure and reopening on the mechanical behaviour should be taken into account when developing accurate numerical models for the analysis and design of dry-stack masonry structures.

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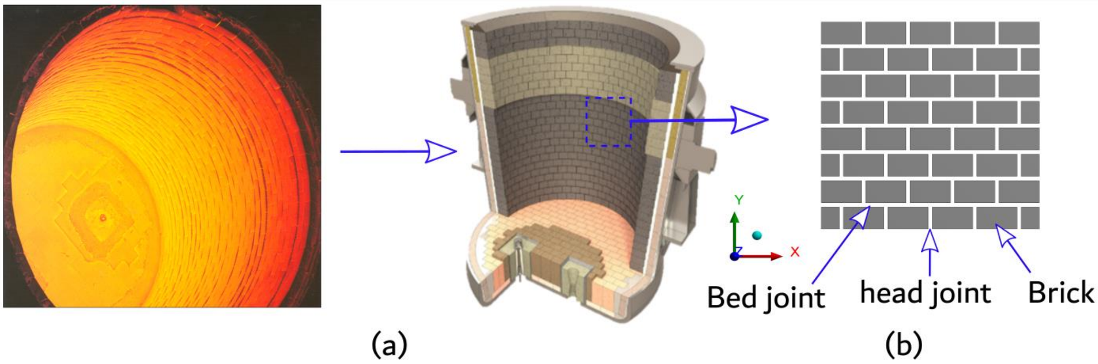

Figure 9 - Steel ladle lined with dry-stack refractory masonry:
(a) section view and (b) schematic of dry-stack masonry structure.

With regard to numerical modelling and homogenization of masonry structures with dry joints, very few numerical studies have been carried out as compared to those of masonry with mortar. Considering all the bricks and joints of the working lining as well as contact between them leads to an increase in the computational time and cost. Furthermore, the convergence of the computation is not guaranteed. For these reasons, a number of authors have neglected the presence of joints [32], while others considered only a few bricks and joints between them, which hereafter will be called micro modelling [33–36]. Also, a multi scale numerical modelling approach can be found in the literature [31].

Regarding micro modelling approach of mortarless masonry, a limited number of studies can be found in the literature. Most of these studies typically focus on studying the impact of surface roughness of the bricks or the bricks' high imperfections on the load bearing capacity of concrete masonry. Usually, the bricks are considered to obey isotropic linear elasticity. For example, a finite element micro model of mortarless block masonry has been developed [37, 38]. The developed micro level model considers dry joints, bricks and interface behaviour (see Figure 10). Dry joints contact behaviour was obtained from experimental data and input into the developed numerical model. The results indicate that the developed FE model is able to accurately predict the deformation of the mortarless blocks subjected to compression loading. Also, dry joints affect the deformation of the blocks significantly from initial loading stage up to 45% of the ultimate loads due to seating impact of dry joints. With the increase of contact area between the blocks, the stiffness of the masonry block increases.

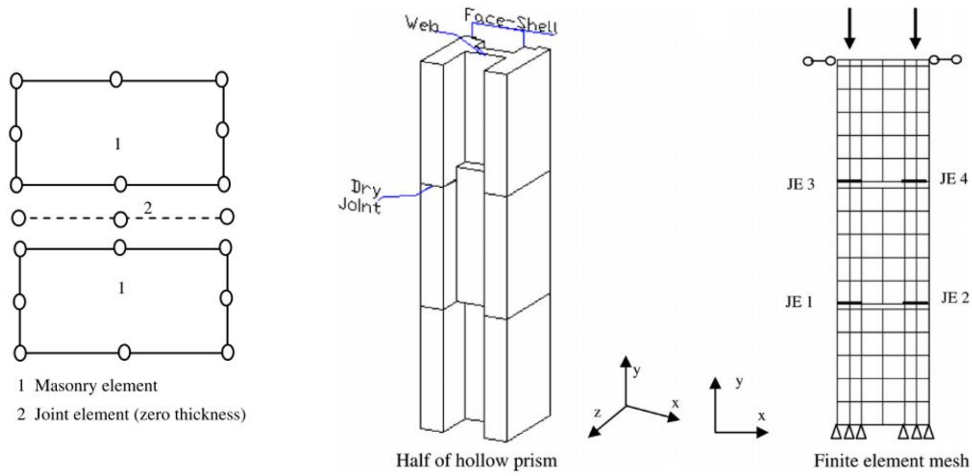

Figure 10 - Drystack masonry model developed in [36]

In the case of stack bond mortarless masonry structures, contact surface roughness adversely impact the constructability of mortarless masonry systems. For this reason, a number of authors carried out experimental and numerical studies to investigate the effects of contact surface roughness on the contact area and pressure in mortarless masonry structures. For example, Zahra and Dhanasekar [33] developed and validated a micro FE model of concrete masonry blocks to study the impact of surface roughness on contact area and pressure (see Figure 11). The charcateristics of contact surfaces and high pressure locations were identified experimentally using pressure surface sensors. Then a micro FE model has been developed taking into account the high pressure loacations. There results indicate that contact area can be increased considerably and, therfore, contact pressure can be decreased by grinding the contact surfaces or by using an auxetic fabric between the contacting surfaces.

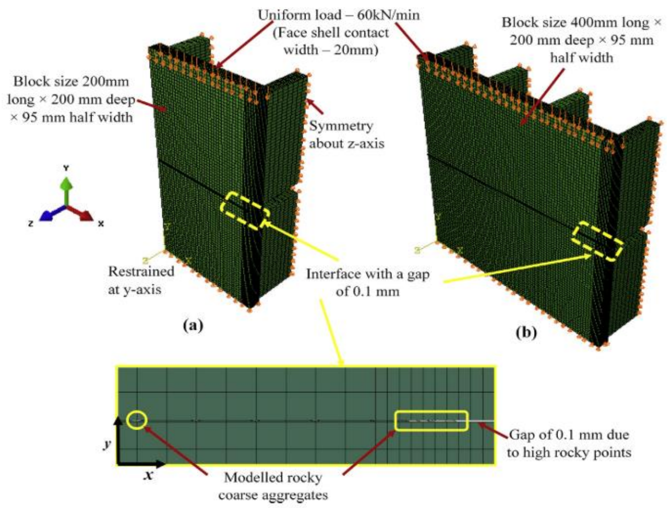

Figure 11 - Micro finite element model of bi-stacked drystack masonry with rough contact surface developed by [33]

A reasonable approach to consider the presence of joints and their impact on the mechanical response of mortarless masonry without increasing computation costs is to replace the bricks and joints by an equivalent material. Nguyen *et al*.[31] developed and validated a homogeneous equivalent material model of mortarless refractory masonry structure. The model considers the influence of joints and joint closure on the mechanical response. This model is based on the four joint patterns presented in Figure 12 as well as the transition criteria between them.

The equivalent elastic properties of each joint pattern have been determined using finite element-based homogenization approach using a Representative Volume Element (RVE) subjected to homogeneous strain boundary conditions. During operation, refractory masonry structure with dry joints is subjected to mechanical or thermal loading and unloading. As a result, bed and/ or head joints may close or open. Therefore, the structure may change from one pattern to another leading to a change in the homogenized elastic behaviour. This change has been considered by using a suitable joint closure and pattern transition criterion written in terms of macroscopic strains (see Figure 12). The developed material model has been implemented in Abaqus using a UMAT subroutine and then used to simulate the mechanical behaviour of mortarless masonry subjected to biaxial compression. Good agreement between experimental and numerical results were observed. Gasser *et al.* [25] used this model and developed a steady-state 3D finite element models of a steel ladle to investigate the influence of bottom design (radial, parallel and fishbone designs) and joint thickness on the resulting thermomechanical stresses. Joints in the ladle bottom have been considered, whereas, joints in the ladle wall have not been taken into account. Their results indicate that using radial design of the ladle bottom results in lower values of Von Mises stress in the steel shell as compared to parallel and fish bone designs. Also, Von Mises stresses in the steel shell decrease with the increase of joint thickness.

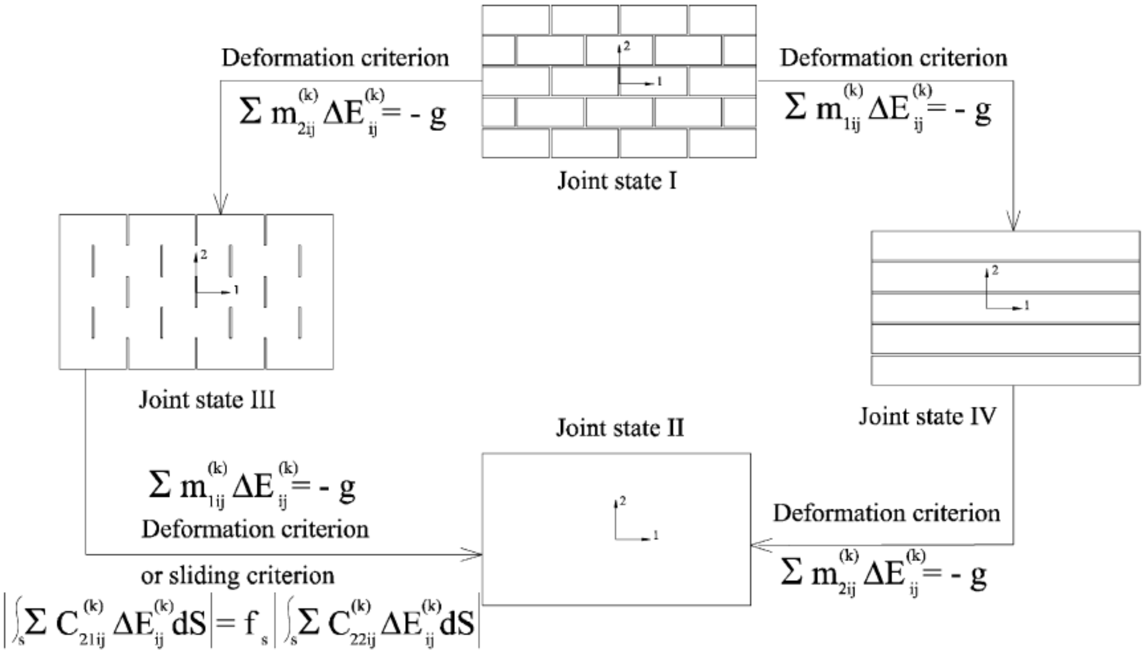

Figure 12 - Possible joint patterns of refractory masonry structure with dry joints and joints closure criteria [30]

4 Possible techniques for modelling homogenized viscoplastic behaviour of masonry

From the previous review, it can be seen that almost all previous studies focused on studying concrete masonry with or without joints and focused on the linear behaviour of the material (i.e. linear elastic). However, at high temperatures (> 1000 °C), the behaviour of refractory masonry structures is nonlinear due to their elastic viscoplastic behaviour. Furthermore, all developed homogenization techniques are only suitable to describe the homogenized elastic damageable behaviour. Below, a nonlinear homogenization technique, that can describe the homogeneous elastic viscoplastic behaviour of periodic heterogeneous solids, is described. Since the structure of masonry is periodic, this technique can be suitable to describe their elastic viscoplastic behaviour at high temperatures.

The orthotropic homogenized steady-state viscoplastic strain rate $(\dot{\bar{\bar{E}}}^{vp})$ of heterogeneous solids can be described using a macroscopic constitutive law developed by Tsuda et al. to study the nonlinear behaviour of plate fin structures used in heat exchangers (See Figure 13) [39, 40]. The main advantage of this constitutive law is that it can describe the orthotropic homogenized viscoplastic behavior of heterogeneous solids using the same parameters as those of the constitutive material.

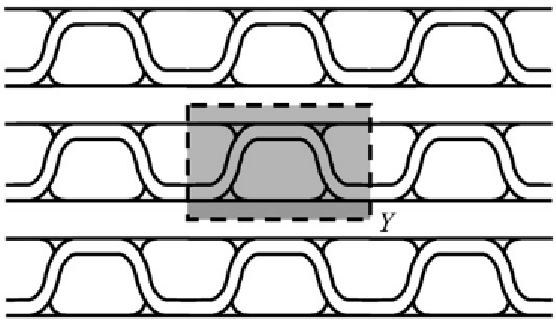

Figure 13 - Periodic plate fin structure and RVE studied by Tsuda et al.[39]

The constitutive law is written as:

$$\dot{\bar{\bar{E}}}^{vp} = \frac{1}{2} A \left(\Sigma_{eq}\right)^{n-1} \bar{\bar{\bar{\bar{\mathbb{N}}}}}\, \bar{\bar{\Sigma}} \qquad (1)$$

With A and n are materials parameters of the bricks, Σ_{eq} and $\overline{\overline{\overline{\overline{\mathbb{N}}}}}$ are the macroscopic equivalent stress and a fourth order rank tensor that accounts for the orthotropy of the structure. The matrix $\overline{\overline{\overline{\overline{\mathbb{N}}}}}$ has 9 non-zero components.

For heterogeneous-orthotropic solids, the macroscopic equivalent stress can be calculated in terms of macroscopic stress and orthotropy matrix ($\overline{\overline{\overline{\overline{\mathbb{N}}}}}$) according to [40, 41]:

$$\Sigma_{eq} = \sqrt{{}^{1}\!/_{2}\,(\overline{\overline{\Sigma}})^{T}\,\overline{\overline{\overline{\overline{\mathbb{N}}}}}\,\overline{\overline{\Sigma}}} \tag{2}$$

Here $(\overline{\overline{\Sigma}})^{T}$ is the transpose of the macroscopic stress second order tensor. Also, The macroscopic equivalent viscoplastic strain rate is defined in terms of $\overline{\overline{\overline{\overline{\mathbb{N}}}}}$ and $\dot{E}^{vp}$ as follows [39, 40]:

$$\dot{E}^{vp}_{eq} = \sqrt{2\left(\overline{\overline{\dot{E}}}^{vp}\right)^{T}\left(\overline{\overline{\overline{\overline{\mathbb{N}}}}}\right)^{-1}\overline{\overline{\dot{E}}}^{vp}} \tag{3}$$

By combining Equations 1, 2 and 3 and using combinations of uni-axial and simple shear finite element numerical tests, the 9 non-zero components of the matrix $\overline{\overline{\overline{\overline{\mathbb{N}}}}}$ can be calculated according to the following equations [39, 40]:

Uniaxial tension a long X – direction:

$$N_{11} = 2\left(\frac{1}{\Sigma_{XX}}\right)^{2}\left(\frac{\Sigma_{XX}\dot{E}^{vp}_{XX}}{A}\right)^{\frac{2}{n+1}} \tag{4}$$

$$N_{12} = \left(\frac{\dot{E}^{vp}_{YY}}{\dot{E}^{vp}_{XX}}\right)N_{11} \tag{5}$$

$$N_{13} = \left(\frac{\dot{E}^{vp}_{ZZ}}{\dot{E}^{vp}_{XX}}\right)N_{11} \tag{6}$$

Uniaxial tension a long Y – direction:

$$N_{22} = 2\left(\frac{1}{\Sigma_{YY}}\right)^{2}\left(\frac{\Sigma_{YY}\dot{E}^{vp}_{YY}}{A}\right)^{\frac{2}{n+1}} \tag{7}$$

$$N_{12} = \left(\frac{\dot{E}^{vp}_{XX}}{\dot{E}^{vp}_{YY}}\right)N_{22} \tag{8}$$

$$N_{23} = \left(\frac{\dot{E}^{vp}_{ZZ}}{\dot{E}^{vp}_{YY}}\right)N_{22} \tag{9}$$

Uniaxial tension along Z-direction:

$$N_{33} = 2\left(\frac{1}{\Sigma_{ZZ}}\right)^{2}\left(\frac{\Sigma_{ZZ}\dot{E}^{vp}_{ZZ}}{A}\right)^{\frac{2}{n+1}} \tag{10}$$

$$N_{13} = \left(\frac{\dot{E}^{vp}_{XX}}{\dot{E}^{vp}_{ZZ}}\right)N_{33} \tag{11}$$

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$$N_{23} = \left(\frac{\dot{E}_{YY}^{vp}}{\dot{E}_{ZZ}^{vp}}\right) N_{33} \tag{12}$$

Simple shear in XY plane:

$$N_{44} = 2\left(\frac{1}{\Sigma_{XY}}\right)^2 \left(\frac{\Sigma_{XY}\dot{E}_{XY}^{vp}}{A}\right)^{\frac{2}{n+1}} \tag{13}$$

Simple shear in XZ plane:

$$N_{55} = 2\left(\frac{1}{\Sigma_{XZ}}\right)^2 \left(\frac{\Sigma_{XZ}\dot{E}_{XZ}^{vp}}{A}\right)^{\frac{2}{n+1}} \tag{14}$$

Simple shear in YZ plane:

$$N_{66} = 2\left(\frac{1}{\Sigma_{YZ}}\right)^2 \left(\frac{\Sigma_{YZ}\dot{E}_{YZ}^{vp}}{A}\right)^{\frac{2}{n+1}} \tag{15}$$

An extension of homogenization models developed by Nguyen et al., 2009 and Brulin et al., 2011 to account for the nonlinear behavior of refractory masonry at high temperature is currently being under development at University of Orléans. The homogenized viscoplastic behavior is described by previously described constitutive model developed by Tsuda et al. [40].

5 Conclusion

Numerous studies concerning linear homogenization of masonries at room temperature can be found in the literature. But only few exist about non linear homogenization at high temperature. The non-linear homogenization methods are currently under development. They are developed and/or adapted, in WP3, to refractory masonries to take into account their elastic viscoplastic behaviour at high temperature.

6 References

[1] S. Russo and F. Sciarretta, ‘Masonry exposed to high temperatures: Mechanical behaviour and properties - An overview’, *Fire Safety Journal*. 2013, doi: 10.1016/j.firesaf.2012.10.001.

[2] P. B. Lourenço and T. Delft, ‘Computational strategies for masonry structures’, 1996.

[3] P. B. Lourenço, ‘Computational strategies for masonry structures’, PhD-Thesis, TU Delft, 1996.

[4] P. B. Lourenço, J. Rots, and J. Blaauwendraad, ‘Two approaches for the analysis of masonry structures’, *Heron*, vol. 40, no. 4, pp. 313–340, 1995.

[5] A. Zucchini and P. B. Lourenço, ‘A micro-mechanical model for the homogenisation of masonry’, *Int. J. Solids Struct.*, vol. 39, no. 12, pp. 3233–3255, 2002, doi: 10.1016/S0020-7683(02)00230-5.

[6] D. Besdo, ‘Inelastic behaviour of plane frictionless block systems described as Cosserat media’, *Arch. Mech.*, vol. 37, no. 6, pp. 603–619, 1985.

[7] H. B. Muhlhaus, ‘Continuum models for layered and blocky rock’, *Compr. Rock Eng. Vol 2*, pp. 209–230, Jan. 1993, doi: 10.1016/b978-0-08-040615-2.50014-9.

[8] A. Anthoine, ‘Derivation of the in-plane elastic characteristics of masonry through homogenization theory’, *Int. J. Solids Struct.*, vol. 32, no. 2, pp. 137–163, Jan. 1995, doi: 10.1016/0020-7683(94)00140-R.

[9] A. Anthoine, ‘Homogenization of periodic masonry: plane stress, generalized plane strain or 3d modelling?’, *Commun. Numer. Methods Eng.*, vol. 13, no. 5, pp. 319–326, May 1997, doi: 10.1002/(SICI)1099-0887(199705)13:5<319::AID-CNM55>3.0.CO;2-S.

[10] A. Urbanski, J. Szarlinski, and Z. Kordecki, ‘Finite element modeling of the behavior of the masonry walls and columns by homogenization approach’, *Comput. Methods Struct. Mason.*, 1995.

[11] G. N. Pande, J. X. Liang, and J. Middleton, ‘Equivalent elastic moduli for brick masonry’, *Comput. Geotech.*, vol. 8, no. 3, pp. 243–265, Jan. 1989, doi: 10.1016/0266-352X(89)90045-1.

[12] G. Maier, A. Nappi, and E. Papa, ‘On damage and failure of brick masonry’, *Exp. Numer. Methods Earthq. Eng.*, 1991.

[13] S. Pietruszczak and X. Niu, ‘A mathematical description of macroscopic behaviour of brick masonry’, *Int. J. Solids Struct.*, vol. 29, no. 5, pp. 531–546, Jan. 1992, doi: 10.1016/0020-7683(92)90052-U.

[14] P. B. Lourenço and J. G. Rots, ‘Multisurface interface model for analysis of masonry structures’, *J. Eng. Mech.*, vol. 123, no. 7, pp. 660–668, 1997, doi: 10.1061/(ASCE)0733-9399(1997)123:7(660).

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[15] A. Zucchini and P. B. Lourenço, 'A coupled homogenisation-damage model for masonry cracking', *Comput. Struct.*, vol. 82, no. 11–12, pp. 917–929, May 2004, doi: 10.1016/j.compstruc.2004.02.020.
[16] A. Zucchini and P. B. Lourenço, 'Mechanics of masonry in compression: Results from a homogenisation approach', *Comput. Struct.*, vol. 85, no. 3–4, pp. 193–204, Feb. 2007, doi: 10.1016/j.compstruc.2006.08.054.
[17] G. Milani, P. B. Lourenço, and A. Tralli, 'Homogenised limit analysis of masonry walls, Part I: Failure surfaces', *Comput. Struct.*, vol. 84, no. 3–4, pp. 166–180, 2006, doi: 10.1016/j.compstruc.2005.09.005.
[18] A. Zucchini, P. B. Lourenço, and A. Tralli, 'Homogenised limit analysis of masonry walls. Part II: structural examples', *Comput. Struct.*, vol. 84, no. 3–4, pp. 181–195, 2006, doi: 10.1016/j.compstruc.2006.08.054.
[19] E. Bertolesi, G. Milani, and P. B. Lourenço, 'Implementation and validation of a total displacement non-linear homogenization approach for in-plane loaded masonry', *Comput. Struct.*, 2016, doi: 10.1016/j.compstruc.2016.08.001.
[20] L. F. Ramos and P. B. Lourenco, 'Characterization of cyclic behavior of dry masonry joints', *J. Struct. Eng.*, 2004, doi: 10.1061/(ASCE)0733-9445(2004)130.
[21] P. B. Lourenço, D. V. Oliveira, P. Roca, and A. Orduña, 'Dry joint stone masonry walls subjected to in-plane combined loading', *J. Struct. Eng.*, vol. 131, no. 11, pp. 1665–1673, 2005, doi: 10.1061/(ASCE)0733-9445(2005)131:11(1665).
[22] R. Senthivel, P. B. Lourenco, and G. Vasconcelos, 'Analytical modeling of dry stone masonry wall under monotonic and reversed cyclic loading', *Struct. Anal. Hist. Constr.*, no. June 2014, 2006.
[23] A. Gasser, K. Terny-Rebeyrotte, and P. Boisse, 'Modelling of joint effects on refractory lining behaviour', *Proc. Inst. Mech. Eng. Part J. Mater. Des. Appl.*, vol. 218, no. 1, pp. 19–28, 2004.
[24] T. M. H. Nguyen, E. Blond, A. Gasser, and T. Prietl, 'Mechanical homogenisation of masonry wall without mortar', *Eur. J. Mech.-ASolids*, vol. 28, no. 3, pp. 535–544, 2009.
[25] A. Gasser *et al.*, 'Influence of different masonry designs of bottom linings', in *Proceedings of the Unified International Technical Conference on Refractories, UNITECR 2013*, 2014, pp. 851–856, doi: 10.1002/9781118837009.ch145.
[26] E. Blond, G. Alain, L. Matthieu, and J.-L. Daniel, 'Modelling of joint effect in refractory structures', 2010.
[27] A. Rekik, T. T. N. Nguyen, and A. Gasser, 'Multi-level modeling of viscoelastic microcracked masonry', *Int. J. Solids Struct.*, vol. 81, pp. 63–83, 2016.
[28] A. Rekik and A. Gasser, 'Numerical homogenization model for effective creep properties of microcracked masonry', *Int. J. Solids Struct.*, vol. 102–103, pp. 297–320, 2016, doi: 10.1016/j.ijsolstr.2016.09.028.
[29] J. Brulin, F. Roulet, R. Mcnally, and M. Micollier, 'Latest Evolution in Blast Furnace Hearth thermo-Mechanical Stress Modelling The BF hearth', *InSteelCon*, no. July, pp. 1–7, 2011.
[30] T. M. H. Nguyen, E. Blond, A. Gasser, and T. Prietl, 'Mechanical homogenisation of masonry wall without mortar', *Eur. J. Mech. ASolids*, vol. 28, no. 3, pp. 535–544, May 2009, doi: 10.1016/j.euromechsol.2008.12.003.
[31] S. Yilmaz, 'Thermomechanical Modelling for Refractory Lining of a Steel Ladle Lifted by Crane', *Steel Res. Int.*, vol. 74, no. 8, pp. 485–490, 2003, doi: 10.1002/srin.200300221.
[32] T. Zahra and M. Dhanasekar, 'Characterisation and strategies for mitigation of the contact surface unevenness in dry-stack masonry', *Constr. Build. Mater.*, vol. 169, pp. 612–628, Apr. 2018, doi: 10.1016/j.conbuildmat.2018.03.002.
[33] G. G. Chewe Ngapeya, D. Waldmann, and F. Scholzen, 'Impact of the height imperfections of masonry blocks on the load bearing capacity of dry-stack masonry walls', *Constr. Build. Mater.*, vol. 165, pp. 898–913, 2018, doi: 10.1016/j.conbuildmat.2017.12.183.
[34] G. G. Chewe Ngapeya and D. Waldmann, 'Overcome of bed-joint imperfections and improvement of actual contact in dry-stacked masonry', *Constr. Build. Mater.*, vol. 233, p. 117173, 2020, doi: 10.1016/j.conbuildmat.2019.117173.
[35] A. Hou, S. Jin, H. Harmuth, and D. Gruber, 'A Method for Steel Ladle Lining Optimization Applying Thermomechanical Modeling and Taguchi Approaches', *Jom*, vol. 70, no. 11, pp. 2449–2456, Nov. 2018, doi: 10.1007/s11837-018-3063-1.
[36] W. A. M. Thanoon, A. H. Alwathaf, J. Noorzaei, M. S. Jaafar, and M. R. Abdulkadir, 'Finite element analysis of interlocking mortarless hollow block masonry prism', *Comput. Struct.*, vol. 86, no. 6, pp. 520–528, 2008, doi: 10.1016/j.compstruc.2007.05.022.
[37] W. A. Thanoon, A. H. Alwathaf, J. Noorzaei, M. S. Jaafar, and M. R. Abdulkadir, 'Nonlinear finite element analysis of grouted and ungrouted hollow interlocking mortarless block masonry system', *Eng. Struct.*, 2008, doi: 10.1016/j.engstruct.2007.10.014.
[38] M. Tsuda and N. Ohno, 'Duplex model for homogenized elastic-viscoplastic behavior of plate-fin structures at high temperatures', *Int. J. Plast.*, vol. 27, no. 10, pp. 1560–1576, Oct. 2011, doi: 10.1016/j.ijplas.2010.10.011.
[39] M. Tsuda, E. Takemura, T. Asada, N. Ohno, and T. Igari, 'Homogenized elastic-viscoplastic behavior of plate-fin structures at high temperatures: Numerical analysis and macroscopic constitutive modeling', *Int. J. Mech. Sci.*, vol. 52, no. 5, pp. 648–656, 2010, doi: 10.1016/j.ijmecsci.2009.06.007.
[40] Z. Xue and J. W. Hutchinson, 'Constitutive model for quasi-static deformation of metallic sandwich cores', *Int. J. Numer. Methods Eng.*, vol. 61, no. 13, pp. 2205–2238, 2004, doi: 10.1002/nme.1142.